
Example nº 5

Earth dam

CivilFEM Manual of Advanced Examples

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5 EXAMPLE N° 5: EARTH DAM

5.1 AIM

The aim of this example is to show the combined possibilities of the utilities that CivilFEM provides for:

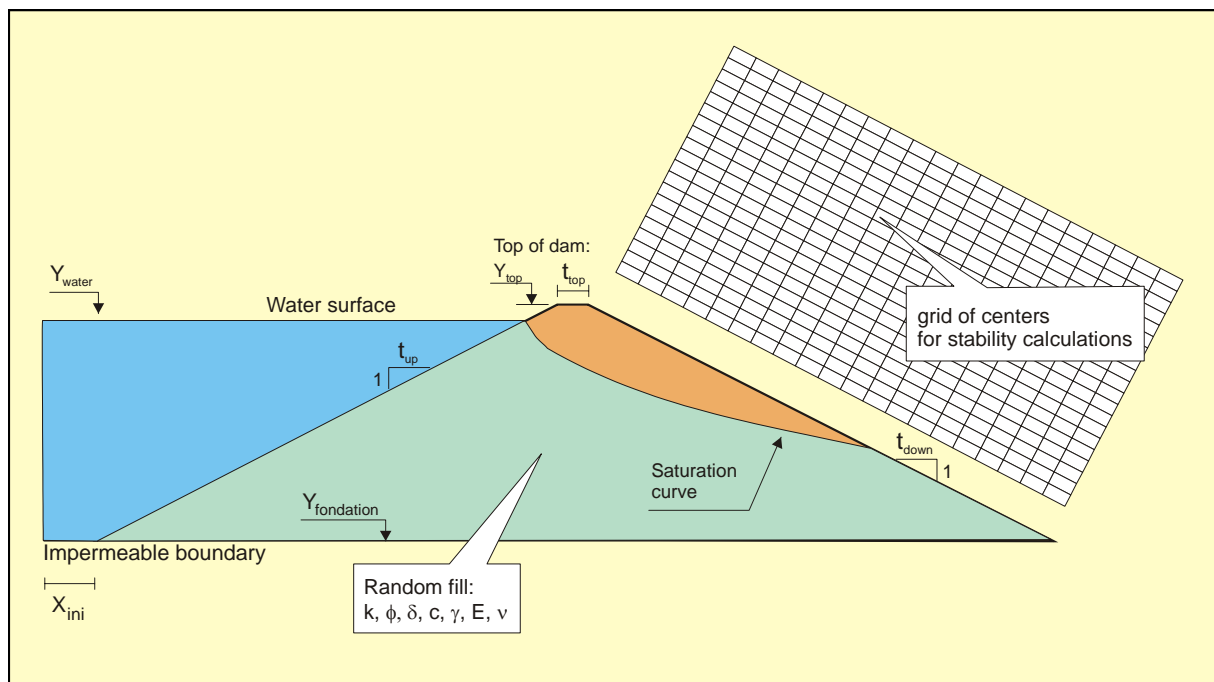
- Seepage analysis, including the calculation of the saturation line.
- Slope stability analysis.

At the same time as it shows its application to successfully complete study of an earth dam, making it possible to the engineer to design the slopes of a dam of these characteristics.

The example shown in this text has a certain complication and, as it will be seen, it requires a lengthy study.

5.2 DESCRIPTION OF THE EXAMPLE

In the following figure the geometry of the dam and the materials characteristics are shown.



The geometric data of the model are:

Yfoundation	=	120.00 m
Ywater	=	190.00 m
Ytop	=	192.00 m
Yexit	=	Yfoundation + 20.00 m
tup	=	2.00 m
tdown	=	2.10 m
ttop	=	10.00 m
xini	=	20.00 m

The properties of land RANDOM FILL are:

k	=	10^{-7} m/s
d	=	30 °
c	=	2 T/m ²
γ_d	=	2.0 T/m ³
E	=	$2 \cdot 10^4$ T/m ²
ν	=	0.30

5.3 RESULTS TO OBTAIN

1. Saturation line and flow through the dam.
2. Pore pressures distribution.
3. Calculate the safety factor of the downstream slope:
 - 3.1. According to Bishop.
 - 3.2. According to Janbu.
4. Analyse the dam using finite elements method. The hydrostatic pressure will be considered as a pressure on the upstream slope.
5. Calculate the law of vertical effective pressures.
6. Obtain the safety factor, according to the circles used in section 3, using the effective pressures obtained in the finite elements method.

5.4 CALCULATION LOG

5.4.1 Introduction

At the beginning of the resolution of **section 1** only the geotechnical materials with low permeability are defined, since the aim of this example is to perform a seepage analysis. The values of these permeabilities are modified (K_x , K_y , K_z).

The parameters that define the geometry of the model are set. Next, the model is created, without boolean operations and with a structure of down to top (Keypoints → Lines → Areas → Elements and nodes).

The saturation line is then defined. It is important to establish the type of boundary condition of the starting and ending points of this line. In this case, the starting point stays fixed at the same location (*FIXED*) as the water level, and the ending point slides along the downstream slope with a seepage condition (*EXIT*).

Once the saturation line is set, the finite elements model is generated from a group of lines, meshing the areas these lines define with *PLANE55* or *PLANE42* elements (in the example *PLANE42* elements are used).

PLANE42 elements (structural 2D solids) will be used if this type of element has been previously defined (*ET* command). In any other case *PLANE55* elements (thermal 2D solid) will be used. The degree of freedom for this calculation is the hydraulic potential H , that CivilFEM will internally associate with the displacement in the X direction (UX) for *PLANE42* elements or with the temperature ($TEMP$) for *PLANE55* elements.

The seepage boundary conditions are then set. These conditions are the hydraulic head upstream and a seepage condition downstream.

The model is finally solved and the results of the flow are obtained.

In **section 2** the pore pressures contour lines are calculated. This simply consists in obtaining a new result from the previous calculations.

It is in this step when the results of pore pressures are transferred to the slope stability utility that will be later used. For this, a file is generated (*jobname.PRESS*) that contains the pore pressures results at each node of the model.

Because of the similarity between the different sections of this example, and in order to simplify the code, in the following sections an *APDL* macro is created. At the beginning of this macro the structural materials are defined, changing their properties to those of the specifications of the example. It is not necessary to modify the permeability parameters of these materials, because they are only going to be used in structural analysis and slope stability calculations. A structural quadratic 2D element (*PLANE82*) with plane strain is used for the calculations. Geometrical parameters are also defined, and from these the keypoints are created. Areas are generated from the keypoints and finally these areas are meshed.

In **section 3** a slope stability analysis is requested by means of two classic methods: Bishop and Janbu.

The classical methods study the stability of a given sliding surface using the Mohr-Coulomb theory and the static equilibrium of moments or forces (or both). The methods compare the destabilizing actions and the stabilizing ones, and they obtain a ratio which is the safety factor.

Once the model is created, the pressures acting on the surface are applied. Later the model is captured to perform the slope stability analysis; to do so 2 points that define the outer surface (the part of the model that may slide) and an intermediate point to show which part of the boundary is the one selected.

Bishop:

The grid of centres of the circles that will be used as sliding surfaces are entered by coordinates or by keypoints, and also the number of divisions at each side. Next, the points that define the mesh of tangents that will define the radii of these circles are also set.

The calculation of the safety factor is an iterative. The parameters that control it (tolerance, maximum number of iterations) can be changed if needed, but in this example the default values will be used.

The pore pressure values used are the ones obtained from the seepage analysis. These values are read from the nodal data included in the *jobname.PRESS* file.

Janbu:

The sliding surfaces definition in this method is different from the definition for Bishop's method. In this case it is necessary to define the points that will set the polygonal sliding surfaces.

The polygonal line has to be convex and must begin and end outside the model (CivilFEM will discard invalid polygonals).

In **section 4** The actions on the model are self weight and the water pressure on the slope, when this one is submerged. Water pressures must be entered as a an external pressure.

For **section 5** CivilFEM's utility to import pore pressure results from a seepage analysis will be used. The meshes of the seepage model and the stability analysis or finite element analysis model do not need to be the same.

The effective pressures are calculated in the structural analysis subtracting the pore pressures.

In **section 6**, in order to compare the results given by the classical methods with the ones provided by the finite element analysis, CivilFEM calculates the equivalent safety factor at each point of the sliding surface.

The normal and tangential stresses are obtained from the results of the finite element analysis. The pore pressures distribution is defined in the same way as for the classical methods.

5.4.2 Section 1: Line of saturation and filtered volume (Generation of the model)

```

FINISH
~CFCLEAR,,1

~CFACTIV,GETC,Y      ! Activate geotechnical module

/TITLE, Slope stability analysis

! CivilFEM Setup
~UNITS,SI

/PREP7
! CivilFEM Preprocessor
! -----
! Materials
~CFMP,1,LIB,SOIL,,SM           ! Layer 1   Sand Silty
~CFMP,1,SOIL,KX,,1e-007
~CFMP,1,SOIL,KY,,1e-007
~CFMP,1,SOIL,KZ,,1e-007

~CFMP,2,LIB,ROCK,,Molena,      ! Layer 2   Molena (Sedimentary Rock)
~CFMP,2,ROCK,KX,,1e-012
~CFMP,2,ROCK,KY,,1e-012
~CFMP,2,ROCK,KZ,,1e-012

! Model definition
! Parameters
Yfoundation = 120.00
Ywater      = 190.00
Ytop        = 192.00
Yexit       = Yfoundation + 20.00
tup         = 2.00
tdown       = 2.10
ttop        = 10.00
Xini        = 20.00
Xleft       = (Ywater - Yfoundation)*tup + Xini
Xleft1      = (Ytop - Yfoundation)*tup + Xini
Xright      = Xleft1 + ttop + (Ytop - Yfoundation)*tdown
Xexit       = Xleft1 + ttop + (Ytop - Yexit)*tdown
XEndPnt     = 200.00
XbaseR      = 500.00

X1 = 0          $ Y1 = 0
X2 = XbaseR     $ Y2 = 0
X3 = 0          $ Y3 = Yfoundation
X4 = Xini       $ Y4 = Y3
X5 = Xright     $ Y5 = Y3
X6 = X2         $ Y6 = Y3
X7 = Xleft      $ Y7 = Ywater
X8 = Xleft1     $ Y8 = Ytop
X9 = X8 + ttop  $ Y9 = Y8
X10 = Xexit     $ Y10 = Yexit

! KP's definition
*DO,II,1,10
  K,II,X%II%,Y%II%
*ENDDO

! Lines
L,1,2
L,2,6
L,1,3
L,3,4

```

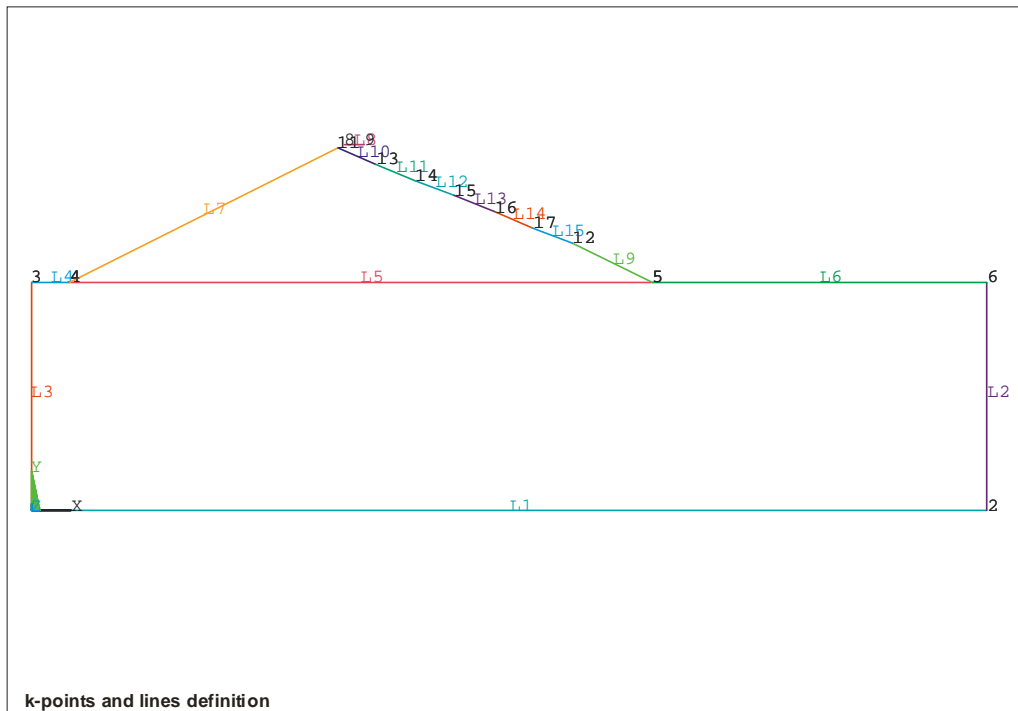
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```
L,4,5
L,5,6
L,4,8
L,8,9
L,9,5
```

```
! Plot lines with their Numbers
/PNUM,KP,1
/PNUM,LINE,1
LPLOT
```

```
! Defines the saturation line
X1      = X7      ! X coordinate of the initial point of the saturation
                ! line (KP,7)
Y1      = Y7      ! Y initial point
Z1      = 0        ! Z initial point
X2      = X10     ! X coordinate of the end point of the saturation line
Y2      = Y10     ! Y end point
Z2      = 0        ! Z end point
NPOINTS = 5       ! Number of points to be generated along the saturation
                ! line
Line1   = 7       ! The line number on which the initial point of the
                ! saturation line leans
Line2   = 9       ! The line number on which the end point of the
                ! saturation line leans
MAT     = 1       ! Material used to mesh the area from which the
                ! saturation line belongs to
```

```
! SATURATION LINE
~WATTAB,X1,Y1,Z1,X2,Y2,Z2,MAT,NPOINTS,Line1,Line2,FIXED,EXIT
/REPLOT
```



```
ET,1,PLANE42
```

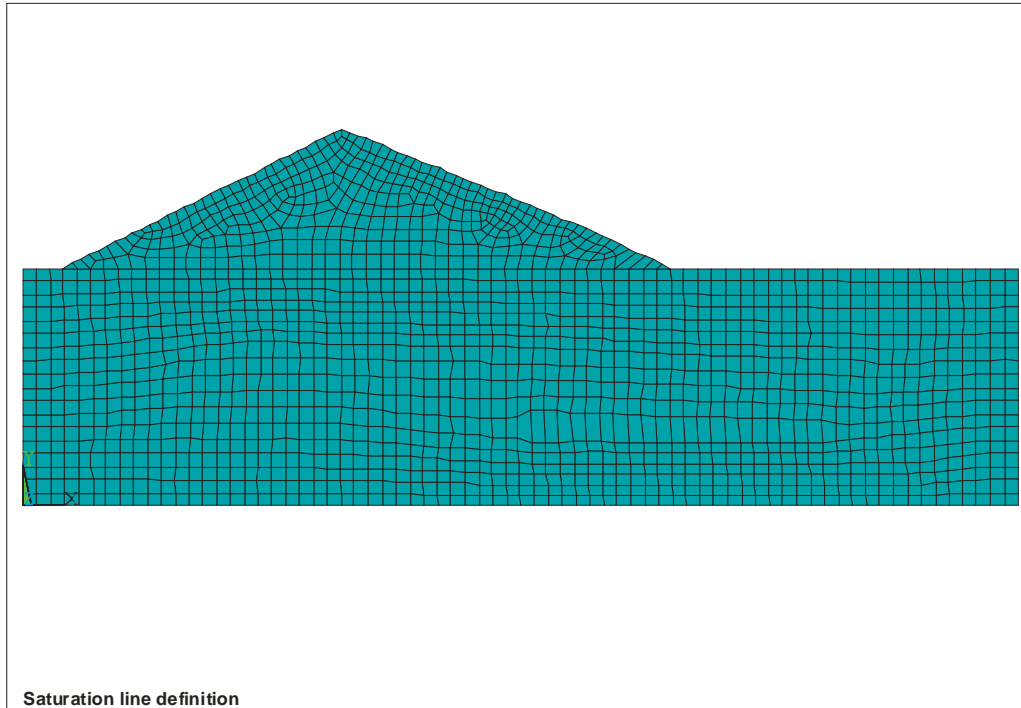
```
! Generate the finite element model from a line set
MAT = 2      ! Material to be used
ESIZE = 7    ! Element size
L1 = 3 $ L2 = 4 $ L3 = 5 $ L4 = 6
L5 = 2 $ L6 = 1
~SEEPROMOD,ADD,MAT,,ESIZE,L1,L2,L3,L4,L5,L6      ! Area 1
```

```

MAT      = 1
ESIZE    = 5
L1       = 7   $   L2       = 5   $   L3       = 9
~SEEPMOD,ADD,MAT,WT,ESIZE,L1,L2,L3           ! Area 3

FINISH

```



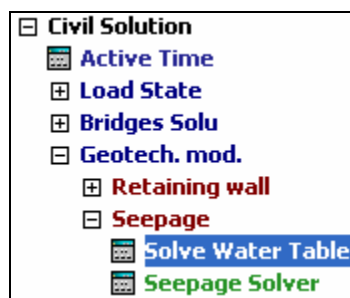
5.4.3 Section 1: Solution

```

/SOLU
! Generates a constant piezometric head as a boundary condition
HEAD1 = Ywater
~DLHEAD,ADD,7,HEAD1
~DLSEEP,ADD,9

! Water Table solver
~WTSOLVE

```



5.4.4 Section 1: Postprocess

```
/POST1
! Calculates the flux through the saturation line
Node1=NODE(194.444,0,0)
Node2=NODE(138.991,159.490,0)
Node3=NODE(131.8176,12.7631,0)
~PLSEEP,Q,GRAPH,N,Node1,Node2,Node3    ! Total flux
~PLSEEP,Q,VECTOR                        ! Vectorial flux
```

5.4.5 Section 2: Pore pressures distribution

```
/GRAPHICS,FULL    ! Display all model geometry and results
~ISOBAR           ! Plot pore water pressure results

! Transfers the pore water pressure results to the slope calculation
! routines
~WTSLP           ! This command automatically generates a text file called
                  ! jobname.PRESS.
```

5.4.6 Section 3.1: Calculation by Bishop's method (model generation)

```
FINISH
~CFCLEAR,,1

~CFACTIV,GETC,Y    ! Activate geotechnical module
/TITLE,Slope stability analysis

! CivilFEM Setup
~UNITS,SI

/PREP7
! CivilFEM Preprocessor
! -----
MACRO_PREP7_FOR_SLOPE_MOD    ! macro that includes: Materials, Element type,
                              ! model & mesh definition

! Boundary Conditions
gammaw = 9810    ! N/m^3
SFL,5,PRES,gammaw*(Y7-Y3),    ! Vertical load over the soil
SFGRAD,PRES,0,Y,Y3,-gammaw    ! Pressure over the slope
SFL,11,PRES,gammaw*(Y7-Y3),
SFTRANS           ! Transfer the solid model surface loads to
                  ! the FEM

/PSF,PRES,NORM,2,0,1    ! Shows surface load symbols on model displays
/PBF,DEFA,,1           ! Shows magnitude of body force loads on
                        ! displays
/PIC,DEFA,,1           ! Shows initial conditions on elements as
                        ! contours on displays
/PSYMB,DOT,1           ! Larger symbols displayed for node and
                        ! keypoint locations

EPLOT

! Define the model contour
~SLPINK,3,6,5          ! Capture the model

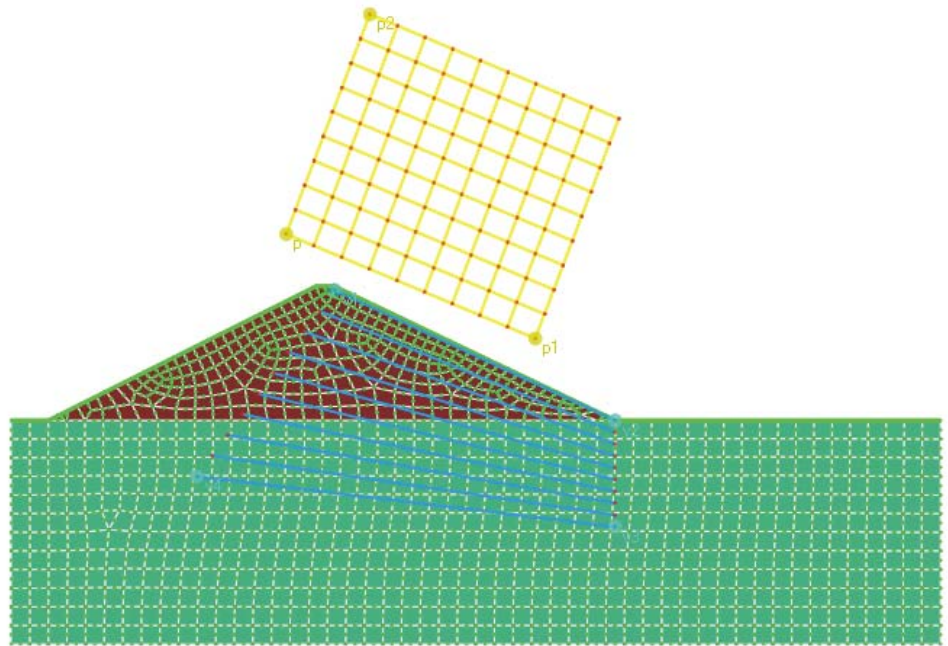
! Introduction of points defining the circle mesh
~SLPCIR,182.0,210.0,282.0,162.0,235.0,302.0,10,10

! Introduction of points defining the tangential mesh
```

```
~SLPTAN,172.0,191.0,325.0,120.0,323.0,100.0,39.0,100.0,10
```

```
~SLPOPT,,,,,3      ! water pressure will be read from jobname.PRESS
                   ! (~WTSLP) in the command ~SLPSOL
```

- [-] Civil Preprocessor
 - [+] Element Types
 - [+] Materials
 - [+] Cross Sections
 - [+] Shell Vertex
 - [+] Member Properties
 - [+] Beam & Shell Properties
 - [+] Solid Section
 - [+] Verif. Database
 - [+] Update Database
 - [+] Active Time
 - [+] Predesign Structures
 - [+] Utilities
 - [+] Load State
 - [+] Bridges Prep
 - [+] Geotech. mod.
 - [+] Terrains
 - [+] EFS
 - [+] Retaining wall
 - [+] Slope stability
 - [+] Define
 - [+] Input Slope Nodes
 - [+] Input Slope KP
 - [+] Graphical input
 - [+] Circles by kp
 - [+] Tangents by kp
 - [+] Foot by kp
 - [+] W-Pressure lines
 - [+] Polyline



5.4.7 Section 3.1: Results

```
/POST1
! Solving slope
~SLPSOL,1,2, ! Bishop Calculation. Slides towards right
```

- [-] Civil Postprocessor
 - [+] Result Summary
 - [+] Alt. to Write
 - [+] Read Results
 - [+] Beam Utilities
 - [+] Shell Utilities
 - [+] Combine Results
 - [+] Envelope Results
 - [+] PreDesign
 - [+] Code Checking
 - [+] Bridge Post
 - [+] Geotech. mod.
 - [+] Slope stability
 - [+] Options
 - [+] Solve
 - [+] Results

5.4.8 Section 3.2: Calculation by Janbu's method (model generation)

```
FINISH
~CFCLEAR,,1
```

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```
~CFACTIV,GETC,Y      ! Activate geotechnical module
/TITLE,Slope establishty analysis

! CivilFEM Setup
~UNITS,SI

/PREP7
! CivilFEM Preprocessor
! -----
MACRO_PREP7_FOR_SLOPE_MOD      ! macro that includes: Materials, Element
                                ! type, model & mesh definition.

! Boundary Conditions
gammaw = 9810  ! N/m^3
SFL,5,PRES,gammaw*(Y7-Y3),      ! Vertical load over the soil
SFGRAD,PRES,0,Y,Y3,-gammaw     ! Pressure over the slope
SFL,11,PRES,gammaw*(Y7-Y3),
SFTRANS                                ! Transfer the solid model surface loads to
                                ! the FEM
/PSF,PRES,NORM,2,0,1            ! Shows surface load symbols on model displays
/PBF,DEFA, ,1                   ! Shows magnitude of body force loads on
                                ! displays
/PIC,DEFA, ,1                   ! Shows initial conditions on elements as
                                ! contours on displays
/PSYMB,DOT,1                    ! Larger symbols displayed for node and
                                ! keypoint locations
EPLOT

! Points that define the model contour
~SLPINK,3,6,5                   ! Capture the model

! Defines the points that generate the Janbu's polygonal
~SLPPOL,100,230,200,170,140,130,120,110,80,50,
~SLPPOL,250,230,200,170,140,130,120,110,80,50,
~SLPPOL,470,230,200,170,140,130,120,110,80,50,

~SLPOPT,,,,,3      ! water pressure will be read from jobname.PRESS (~WTSLP)
                   ! in the command ~SLPSOL
```

5.4.9 Section 3.2: Results

```
/POST1
! Solving slope
~SLPSOL,2,2      ! Janbu Calculation. Slides towards right
```

5.4.10 Section 4: Calculation by FEM (model generation)

```
FINISH
~CFCLEAR,,1

/TITLE,FEM analysis

! CivilFEM Setup
~UNITS,SI

/PREP7
! CivilFEM Preprocessor
! -----
! macro that includes: Materials, Element type, model & mesh definition
MACRO_PREP7_FOR_SLOPE_MOD
```

```

! Boundary conditions
DL,1,,UY,0
DL,2,,UX,0
DL,6,,UX,0
DTRAN                      ! Transfers solid model DOF constraints to the FEM
EPLOT

```

5.4.11 Section 4: Calculation

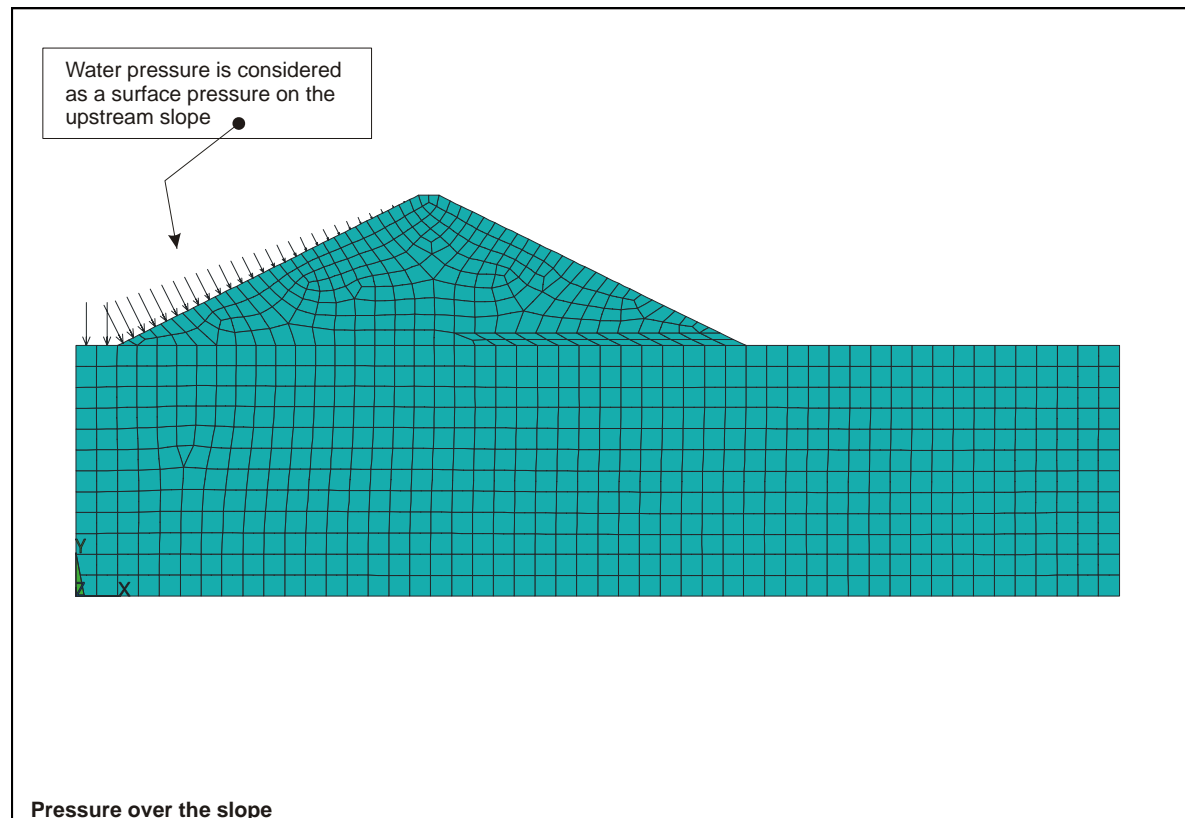
```

/SOLU
! Gravity
ACEL,,9.8                      ! Self Weight

gammaw = 9810  ! N/m^3
SFL,5,PRES,gammaw*(Y7-Y3),      ! Vertical load over the soil
SFGRAD,PRES,0,Y,Y3,-gammaw      ! Pressure over the slope
SFL,11,PRES,gammaw*(Y7-Y3),
SFTRANS                          ! Transfer the solid model surface loads to
                                ! the FEM
/PSF,PRES,NORM,2,0,1            ! Shows surface load symbols on model displays
/PBF,DEFA, ,1                  ! Shows magnitude of body force loads on
                                ! displays
/PIC,DEFA, ,1                  ! Shows initial conditions on elements as
                                ! contours on displays
/PSYMB,DOT,1                   ! Larger symbols displayed for node and
                                ! keypoint locations
EPLOT

! Solving the finite element model
SOLVE

```



5.4.12 Section 4: Postprocess

```
/POST1
~CFSET,,1
/GRAPHICS,FULL          ! Display all model geometry and results
/DSCALE,1,1.0
PLNSOL,S,Y
```

5.4.13 Section 5: Effective pressures

```
~EFFPRES,Y      ! Effective Pressures
```

5.4.14 Section 6: Safety factor

```
FINISH
~CFCLEAR,,1

~CFACTIV,GETC,Y      ! Activate geotechnical module
/TITLE,FEM analysis

! CivilFEM Setup
~UNITS,SI

/PREP7
! CivilFEM Preprocessor
! -----
! macro that includes: Materials, Element type, model & mesh definition
MACRO_PREP7_FOR_SLOPE_MOD

! Boundary conditions
DL,1,,UY,0
DL,2,,UX,0
DL,6,,UX,0
DTRAN                      ! Transfers solid model DOF constraints to the FEM
EPLOT

! Points that define the model contour
~SLPINK,3,6,5      ! Capture the model

! Introduction of points defining the circle mesh
~SLPCIR,182.0,210.0,282.0,162.0,235.0,302.0,10,10

! Introduction of points defining the tangential mesh
~SLPTAN,172.0,191.0,325.0,120.0,323.0,100.0,39.0,100.0,10

~SLPOPT,,,,,3      ! water pressure will be read from jobname.PRESS
                  ! (~WTSLP) in the command ~SLPSOL
```

5.4.15 Section 6: Calculation

The log file is the same as the one used in the section 5.4.11

5.4.16 Section 6: Postprocess

```
/POST1
~CFSET,,1
!Solving slope
~SLPSOL,10,2      ! FEM Calculation. Slides towards right
```

5.4.17 Auxiliary macro

This macro is used in some of the APDL files of the previous sections.

```
! MACRO_PREP7_FOR_SLOPE_MOD: macro that includes: Materials, Element type,
! model & mesh definition.
! Used in slope stability problem 5 sections: BISHOP, JANBU, FEM

! Materials
~CFMP,1,LIB,SOIL,,SM      ! Layer 1   Sand Silty
~CFMP,1,SOIL,EXST,,1.96E8 ! Young's Modulus = 2.10E4 T/m^2 = 1.96E8 N/m^2
~CFMP,1,SOIL,PHIMCEFF,,35 ! PHIMCeff = 35°
~CFMP,1,SOIL,GAMD,,19600  ! GAMD = 2T/m^3 =2*9800N/m^3
~CFMP,1,SOIL,CMCEFF,,19600 ! CMCEFF = 2T/m^2 =2*9800N/m^2
~CFMP,1,SOIL,DELEFF,,35/2

~CFMP,2,LIB,ROCK,,Molena ! Layer 2   Molena (Sedimentary Rock)

! Ansys Preprocessor
ET,1,PLANE82              ! Element type 1: Solid-2D
KEYOPT,1,3,2              ! Plane Strain

! Model definition
! Parameters
Yfoundation = 120.00
Ywater      = 190.00
Ytop        = 192.00
Yexit       = Yfoundation + 20.00
tup         = 2.00
tdown       = 2.10
ttop        = 10.00
Xini        = 20.00
Xleft       = (Ywater - Yfoundation)*tup + Xini
Xleft1      = (Ytop - Yfoundation)*tup + Xini
Xright      = Xleft1 + ttop + (Ytop - Yfoundation)*tdown
Xexit       = Xleft1 + ttop + (Ytop - Yexit)*tdown
XEndPnt     = 200.00
XbaseR      = 500.00

X1 = 0          $ Y1 = 0
X2 = XbaseR     $ Y2 = 0
X3 = 0          $ Y3 = Yfoundation
X4 = Xini       $ Y4 = Y3
X5 = Xright     $ Y5 = Y3
X6 = X2         $ Y6 = Y3
X7 = Xleft      $ Y7 = Ywater
X8 = Xleft1     $ Y8 = Ytop
X9 = X8 + ttop  $ Y9 = Y8
X10 = Xexit     $ Y10 = Yexit

! KP's definition
*DO,II,1,10
  K,II,X%II%,Y%II%
*ENDDO
/PNUM,KP,1
```

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```
/REPLOT
KPLOT

! Generate areas (layers)
A,1,2,6,5,4,3      ! Area 1
A,5,10,9,8,7,4     ! Area 2

! Meshing
AESIZE,ALL,10      ! Element size = 3m

! Mesh layers
MAT,2
AMESH,1            ! Meshing Area 1

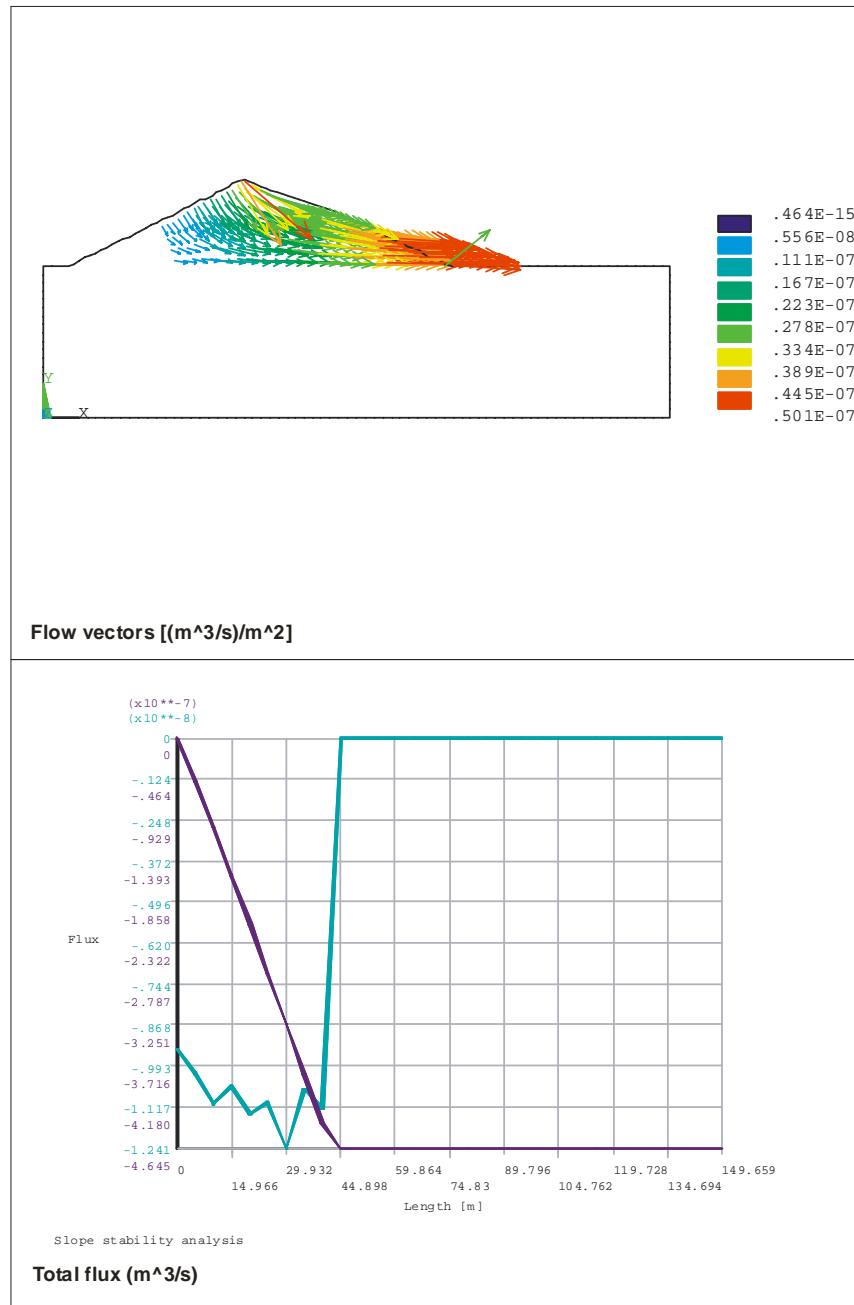
AESIZE,ALL,6
MAT,1
AMESH,2            ! Meshing Area 2

ALLSEL,ALL
EPLOT
```

5.5 RESULTS

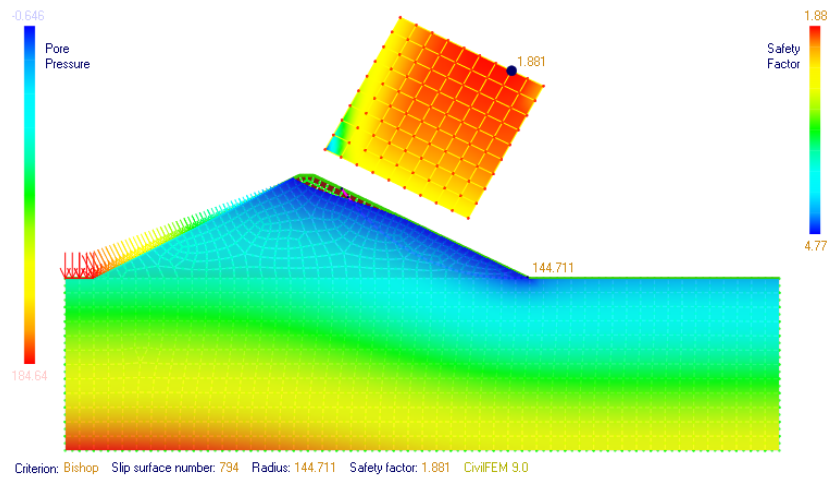
In the following pages graphs with the obtained results are shown.

5.5.1 Seepage through the dam



These results may vary slightly because the free mesh

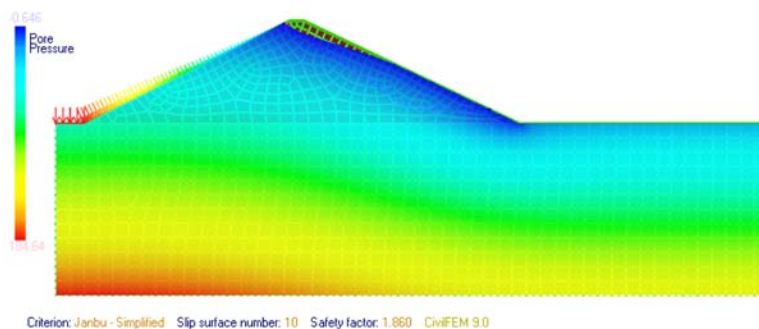
Stability by Bishop's method



A safety factor of 1.8 for the worst sliding surface is obtained.

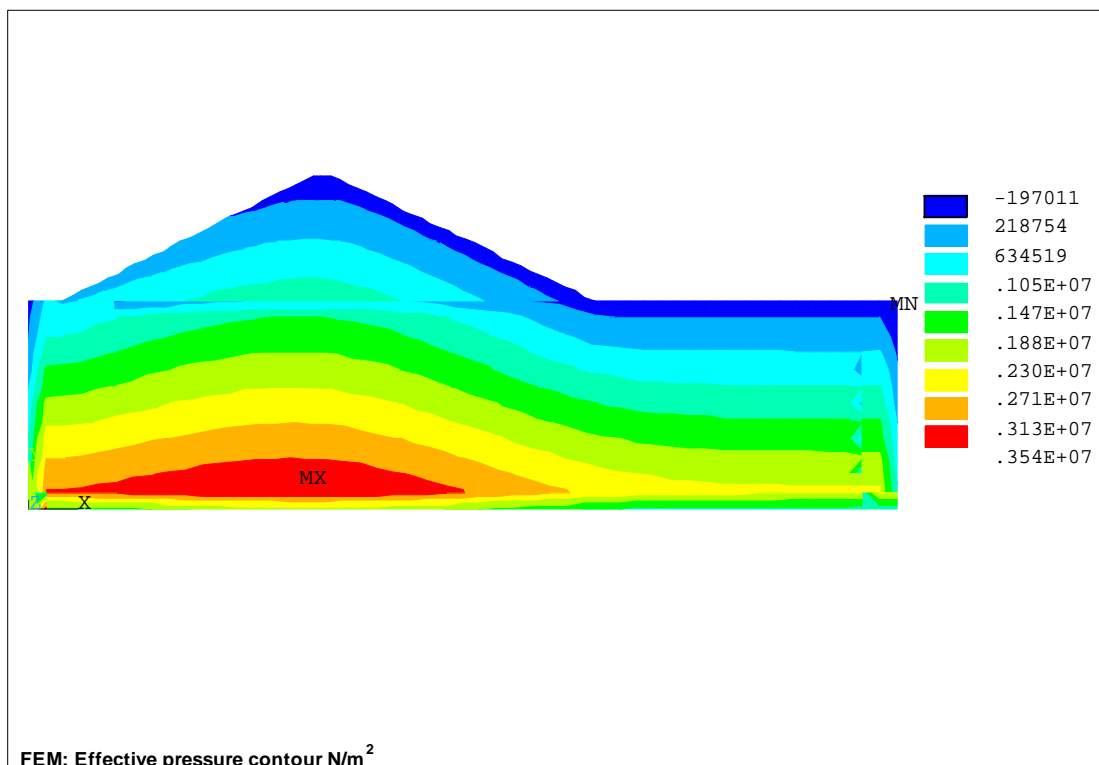
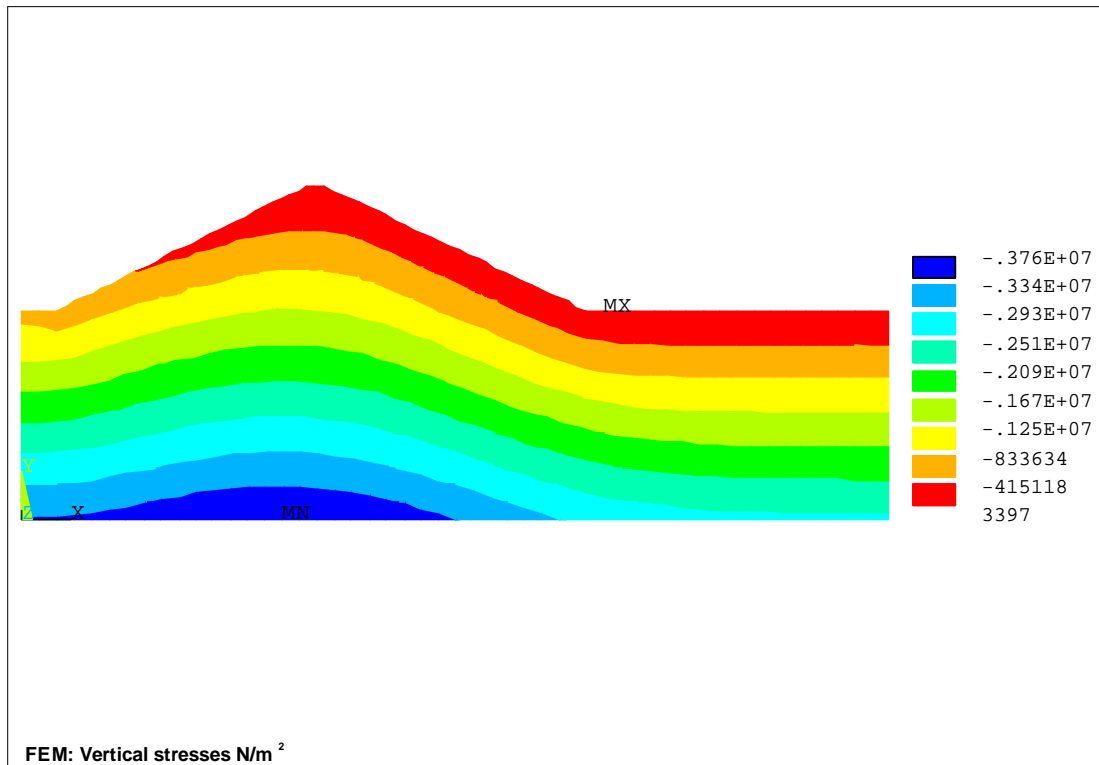
It is important that the point of the center's grid that determines the smaller safety factor is not located in the boundary of this mesh, because this would imply that points located outside this contour could still have lower safety factors. In this example, expanding the grid would not provide significantly smaller results of the safety factor, reason why the obtained safety factor will be considered acceptable.

5.5.2 Stability by Janbu's method

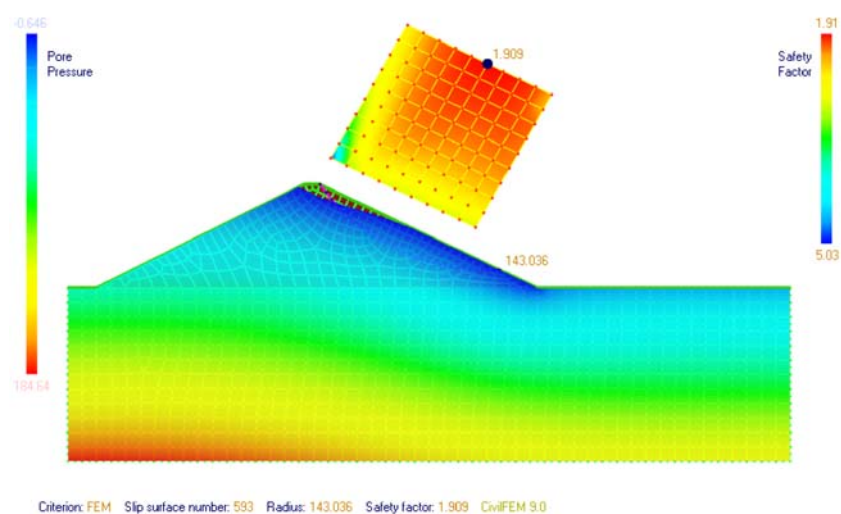


In this plot, the sliding surface can be seen as a violet outline. This sliding surface provides a safety factor of 1.9.

5.5.3 Total stresses and effective pressures (FEM)



5.5.4 FEM Safety Factor



5.5.5 Values comparison

Analysing the obtained results by the different methods, the following values are obtained

Method	F
Bishop	1.8
Janbu	1.9
FEM	1.9

5.6 SUMMARY

With this exercise the user has had the occasion to practice and to understand the way in which CivilFEM treats the following subjects:

- Seepage:
 - Handling of the seepage elements.
 - Obtaining of the saturation line.
 - Calculation of seepage flows.
- Slope stability analysis:
 - Interaction between the seepage analysis and the slope stability calculation.
 - Handling some of the classical methods of stability analysis (Bishop and Janbu).
 - Calculation of the sliding safety factor from the finite elements analysis.

The studied subjects cover a wide range of possibilities, frequently used in civil engineering, particularly in the analysis of earth dams or general slopes.